Neural networks

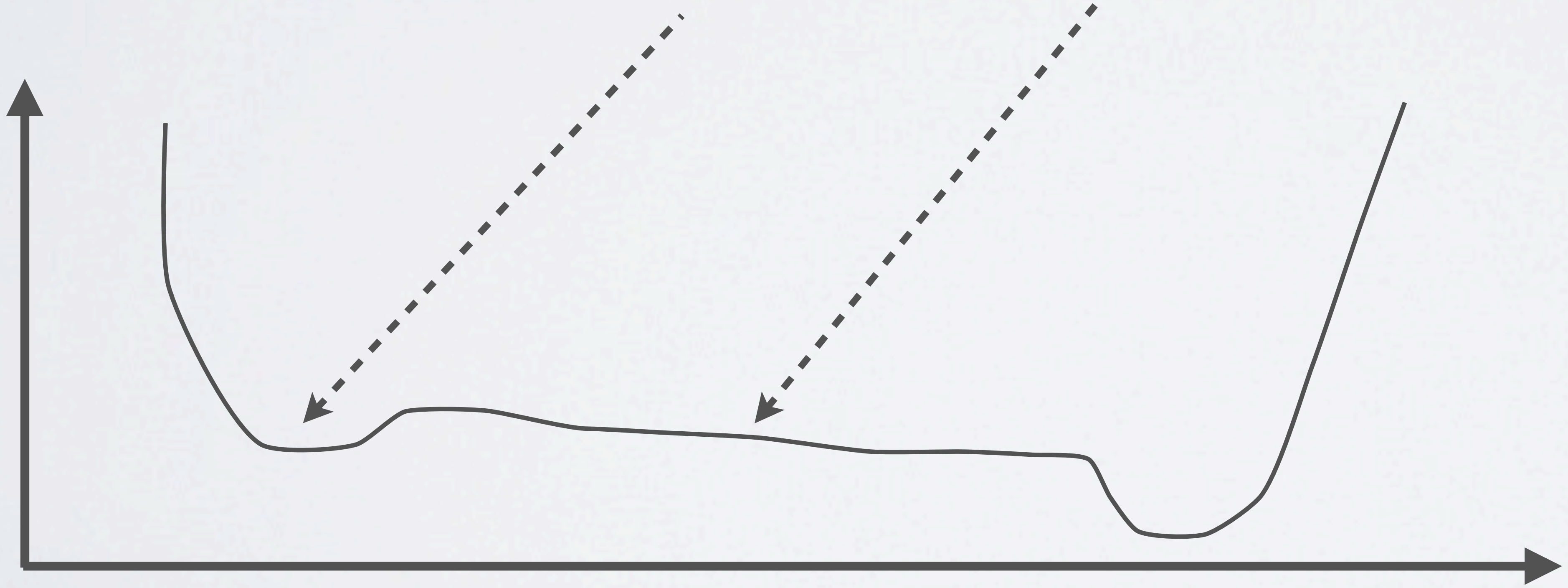
Training neural networks - optimization

OPTIMIZATION

Topics: local optimum, global optimum, plateau

- Notes on the optimization problem
  - there isn't a single global optimum (non-convex optimization)
    - we can permute the hidden units (with their connections) and get the same function
    - we say that the hidden unit parameters are not identifiable

- Optimization can get stuck in local minimum or plateaus



OPTIMIZATION

Topics: local optimum, global optimum, plateau

Neural network training demo
(by Andrej Karpathy)

http://cs.stanford.edu/~karpathy/svmjs/demo/demonn.html

GRADIENT DESCENT

Topics: convergence conditions, decrease constant

- Stochastic gradient descent will converge if
  - $\sum_{t=1}^{\infty} \alpha_t = \infty$
  - $\sum_{t=1}^{\infty} \alpha_t^2 < \infty$

  where α_t is the learning rate of the t^{th} update
- Decreasing strategies: (δ is the decrease constant)
  - $\alpha_t = \frac{\alpha}{1+\delta t}$
  - $\alpha_t = \frac{\alpha}{t^\delta}$ (où $0.5 < \delta \leq 1$)
- Better to use a fixed learning rate for the first few updates

GRADIENT DESCENT

Topics: mini-batch, momentum

- Can update based on a mini-batch of example (instead of 1 example):
  - the gradient is the average regularized loss for that mini-batch
  - can give a more accurate estimate of the risk gradient
  - can leverage matrix/matrix operations, which are more efficient

- Can use an exponential average of previous gradients:

$$\overline{\nabla}_{\boldsymbol{\theta}}^{(t)} = \nabla_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x}^{(t)}), y^{(t)}) + \beta \overline{\nabla}_{\boldsymbol{\theta}}^{(t-1)}$$

  - can get through plateaus more quickly, by "gaining momentum"

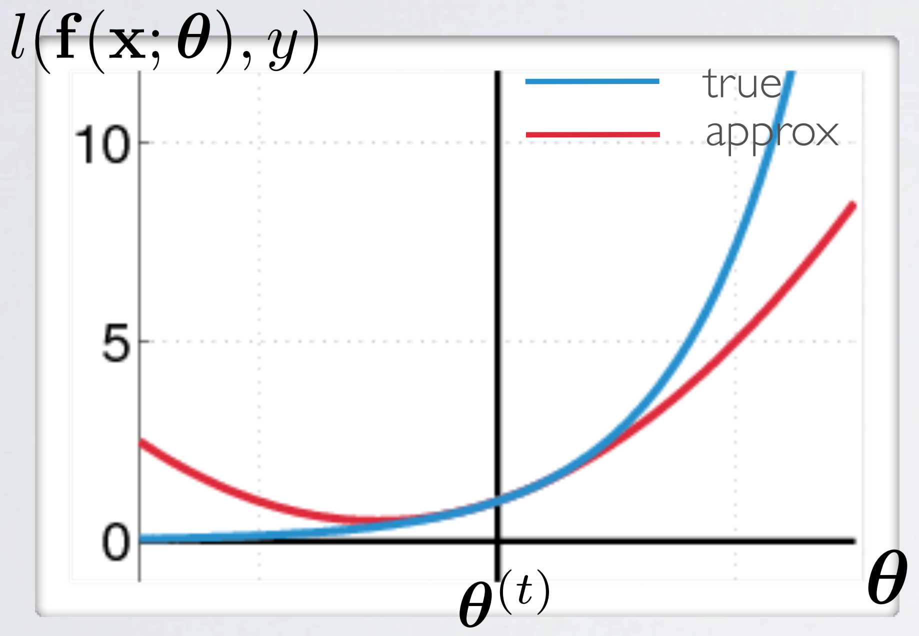
GRADIENT DESCENT

Topics: Newton's method

- If we locally approximate the loss through Taylor expansion:

$$
\begin{aligned}
l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}),y) \quad \approx \quad & l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}),y) + \nabla_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}),y)^\top (\boldsymbol{\theta} - \boldsymbol{\theta}^{(t)}) \\
& +0.5(\boldsymbol{\theta} - \boldsymbol{\theta}^{(t)})^\top \underbrace{\left(\nabla^2_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}),y)\right)}_{\text{Hessian}} (\boldsymbol{\theta} - \boldsymbol{\theta}^{(t)})
\end{aligned}
$$

Hessian



- We could minimize that approximation, by solving:

$$
0 = \nabla_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}),y) + \left(\nabla^2_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}),y)\right)(\boldsymbol{\theta} - \boldsymbol{\theta}^{(t)})
$$

GRADIENT DESCENT

Topics: Newton's method

- We can show that the minimum is:

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \left(\nabla^2_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}), y)\right)^{-1} \left(\nabla_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x};\boldsymbol{\theta}^{(t)}), y)\right)$$

- Only practical if:
  - few parameters (so we can invert Hessian)
  - locally convex (so the Hessian is invertible)

- See recommended readings for more on optimization of neural networks