Neural networks

Conditional random fields - computing marginals

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Topics: computing $p(\mathbf{y}|\mathbf{X})$

- Then we have:

$$p(\mathbf{y}|\mathbf{X}) = \exp\left(\sum_{k=1}^{K} a_u(y_k) + \sum_{k=1}^{K-1} a_p(y_k, y_{k+1})\right) / Z(\mathbf{X})$$

where

$$Z(\mathbf{X}) = \sum_{y'_1} \sum_{y'_2} \cdots \sum_{y'_K} \exp\left(\sum_{k=1}^{K} a_u(y'_k) + \sum_{k=1}^{K-1} a_p(y'_k, y'_{k+1})\right)$$

hard to compute

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Topics: forward/backward or belief propagation

- Computing both tables is often referred to as the forward/backward algorithm for CRFs
  - α is computed with a forward pass
  - β is computed with a backward pass
- It has other names
  - belief propagation
  - sum-product
- α gives the summation from the left
- β gives the summation from the right

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Topics: computing $p(y_k|\mathbf{X}),\ p(y_k,\ y_{k+1}|\mathbf{X})$

- The $\alpha\,/\beta$ tables can be used to compute marginals

$$p(y_k|\mathbf{X}) = \frac{\exp(a_u(y_k) + \log\alpha_{k-1}(y_k) + \log\beta_{k+1}(y_k))}{\sum_{y'_k}\exp(a_u(y'_k) + \log\alpha_{k-1}(y'_k) + \log\beta_{k+1}(y'_k))}$$

$$p(y_k, y_{k+1}|\mathbf{X}) = \frac{\exp\left(\begin{array}{l} a_u(y_k) + a_p(y_k, y_{k+1}) + a_u(y_{k+1}) \\ +\log\alpha_{k-1}(y_k) + \log\beta_{k+2}(y_{k+1}) \end{array}\right)}{\sum_{y'_k}\sum_{y'_{k+1}}\exp\left(\begin{array}{l} a_u(y'_k) + a_p(y'_k, y'_{k+1}) + a_u(y'_{k+1}) \\ +\log\alpha_{k-1}(y'_k) + \log\beta_{k+2}(y'_{k+1}) \end{array}\right)}$$