Neural networks

Conditional random fields - Markov network

LINEAR CHAIN CRF

Topics: factor, sufficient statistic

- With hidden units, the CRF factors could be:

$$\Psi_f(\mathbf{y}, \mathbf{X}) = \begin{cases} \phi_f(y_k, \mathbf{x}_{k-1}) = \exp\left(a^{(L+1,-1)}(\mathbf{x}_{k-1})_{y_k}\right) \\ \phi_f(y_k, \mathbf{x}_k) = \exp\left(a^{(L+1,0)}(\mathbf{x}_k)_{y_k}\right) \\ \phi_f(y_k, \mathbf{x}_{k+1}) = \exp\left(a^{(L+1,+1)}(\mathbf{x}_{k+1})_{y_k}\right) \\ \phi_f(y_k, y_{k+1}) = \exp\left(V_{y_k, y_{k+1}}\right) \end{cases}$$

- There is no simple form for the sufficient statistics

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unary factors

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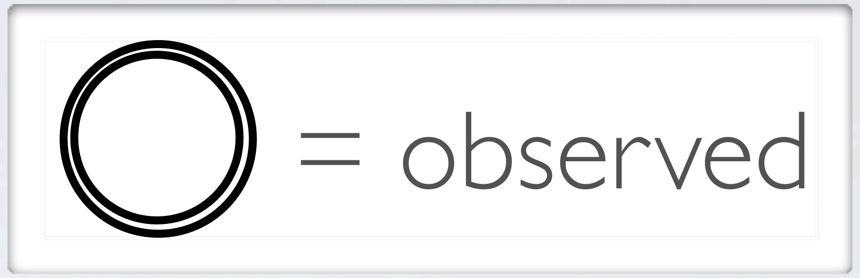
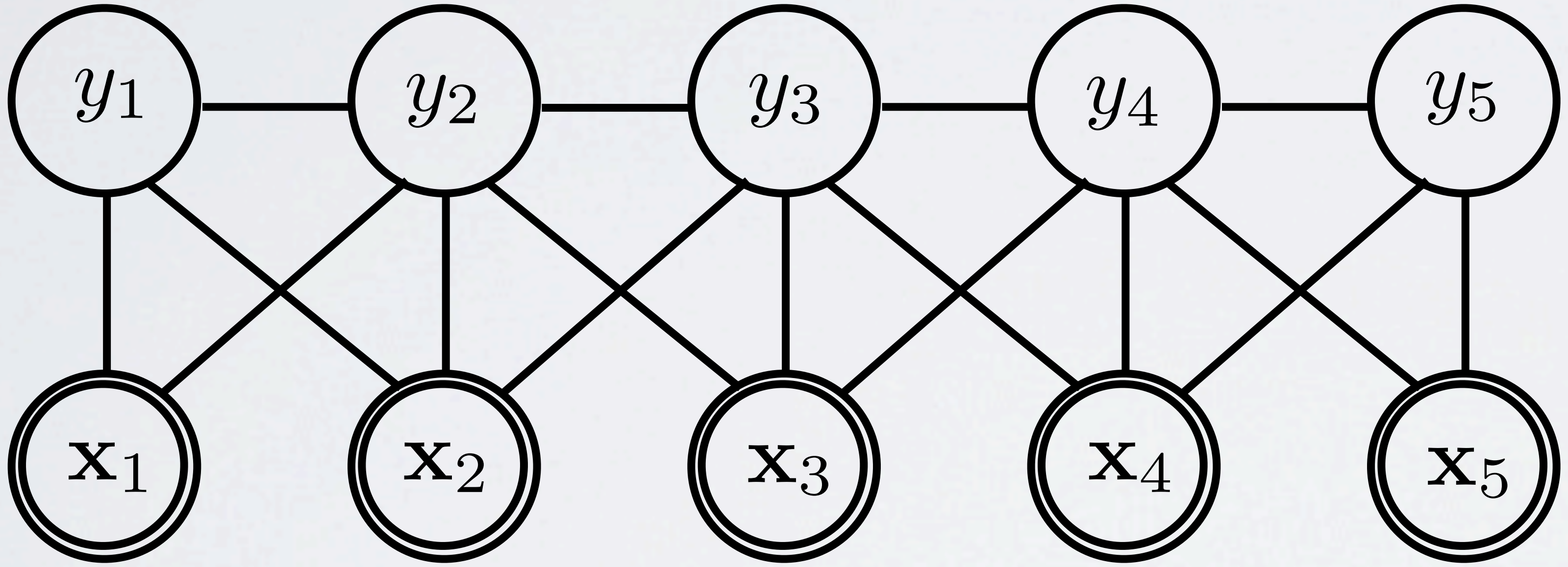
MARKOV NETWORK VISUALIZATION

Topics: Markov network



- Illustration for $K{=}5$



- Network with an edge between each node (random variable) that shares a factor

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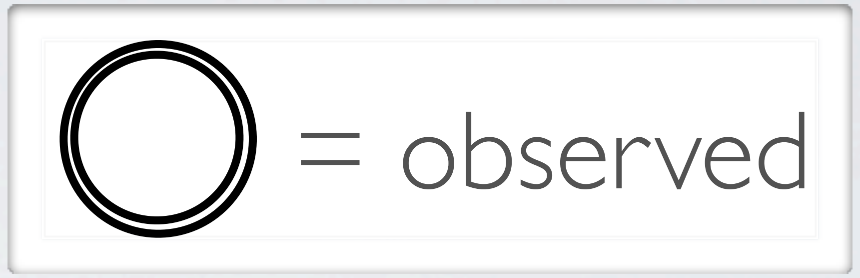

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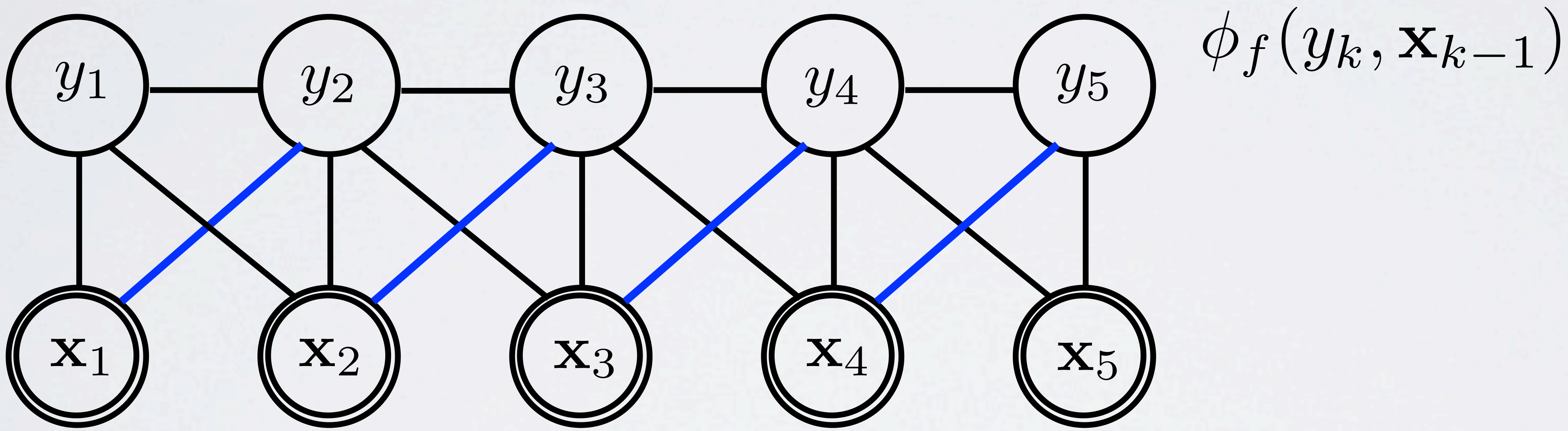

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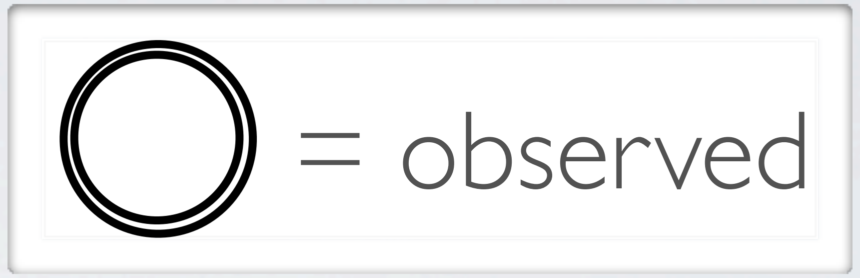

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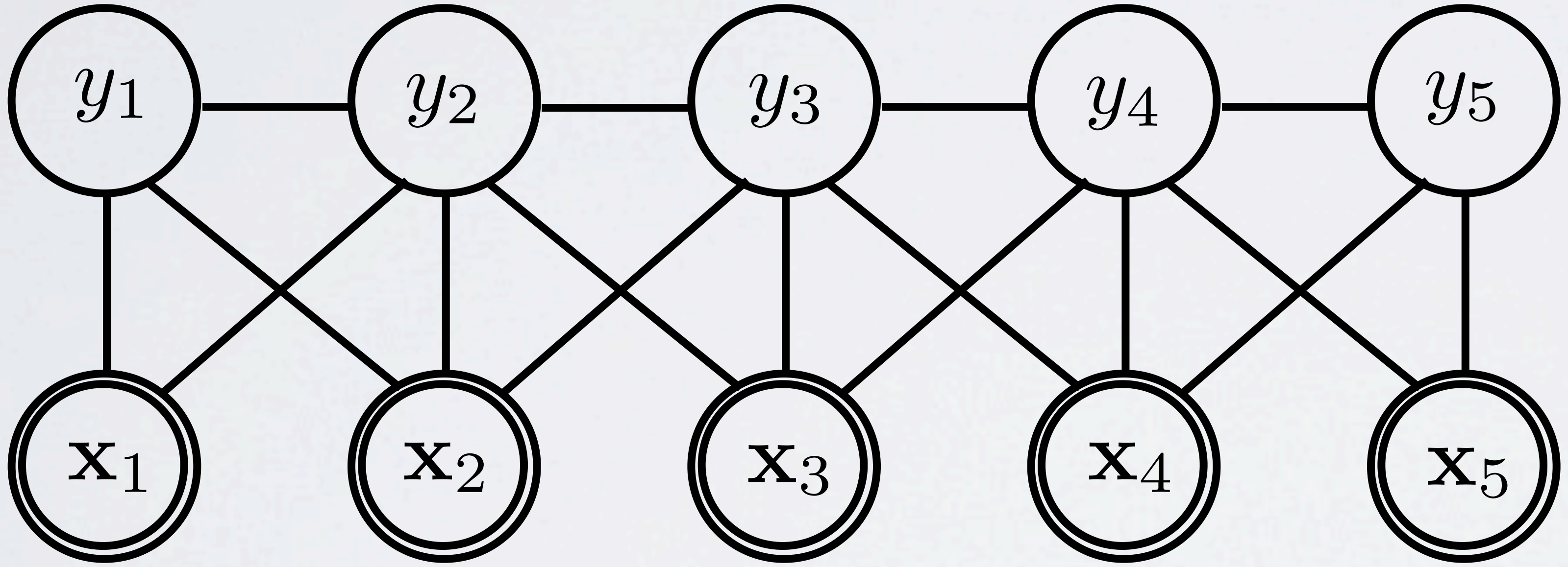

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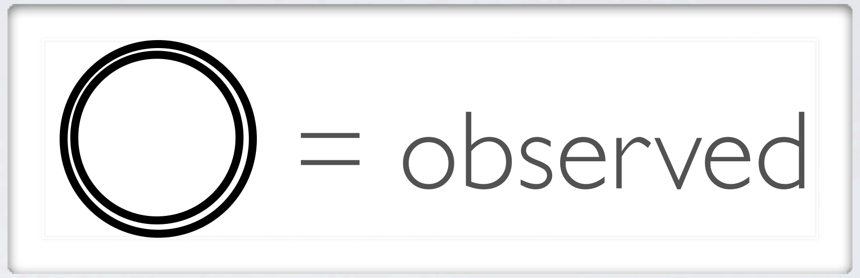

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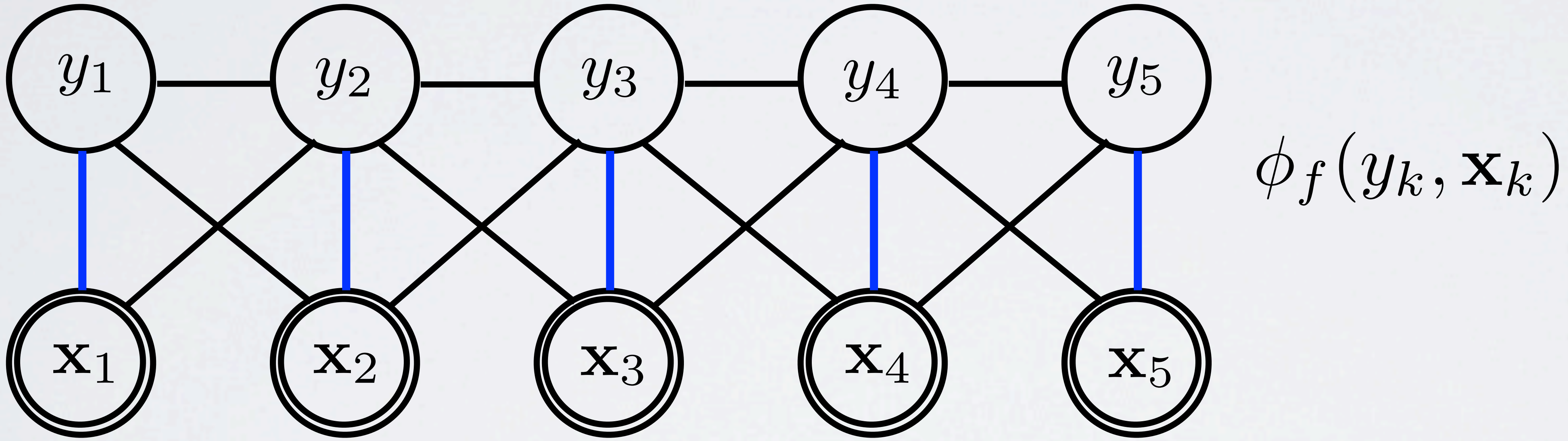

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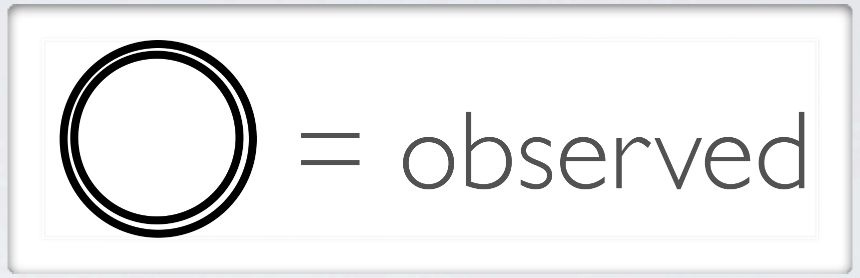

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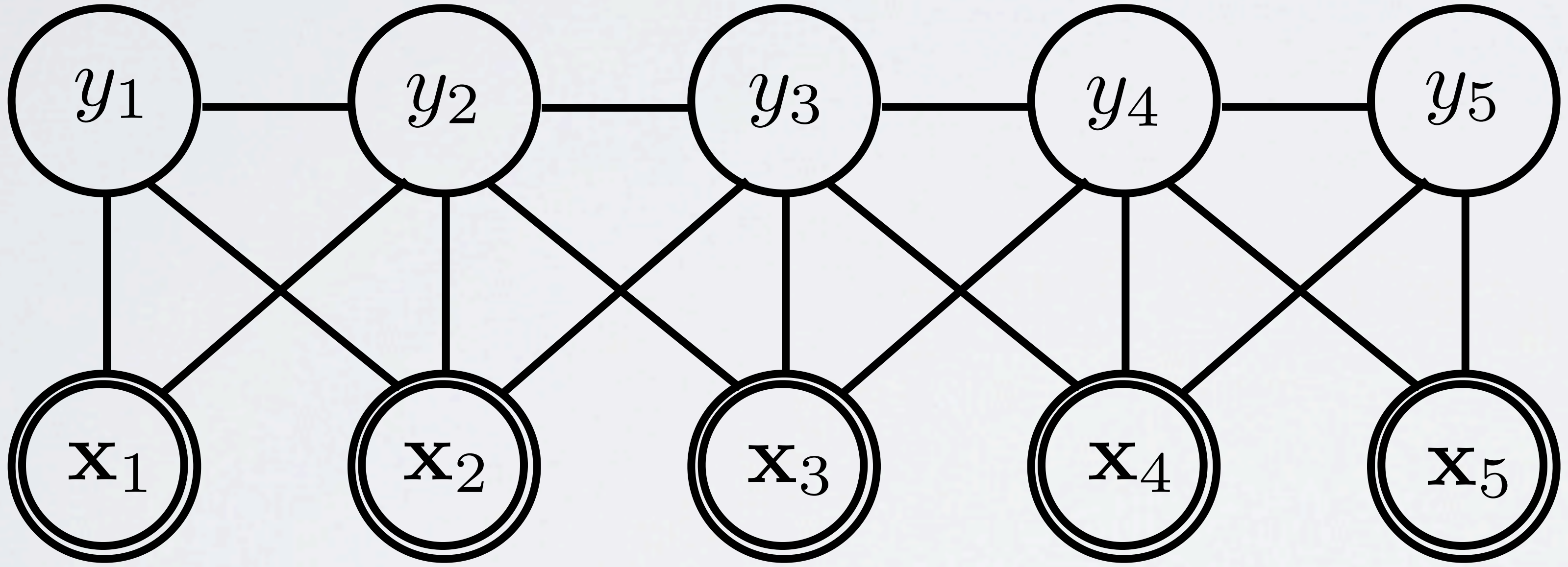

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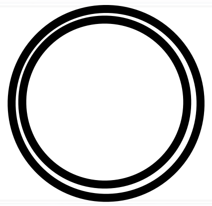 = observed

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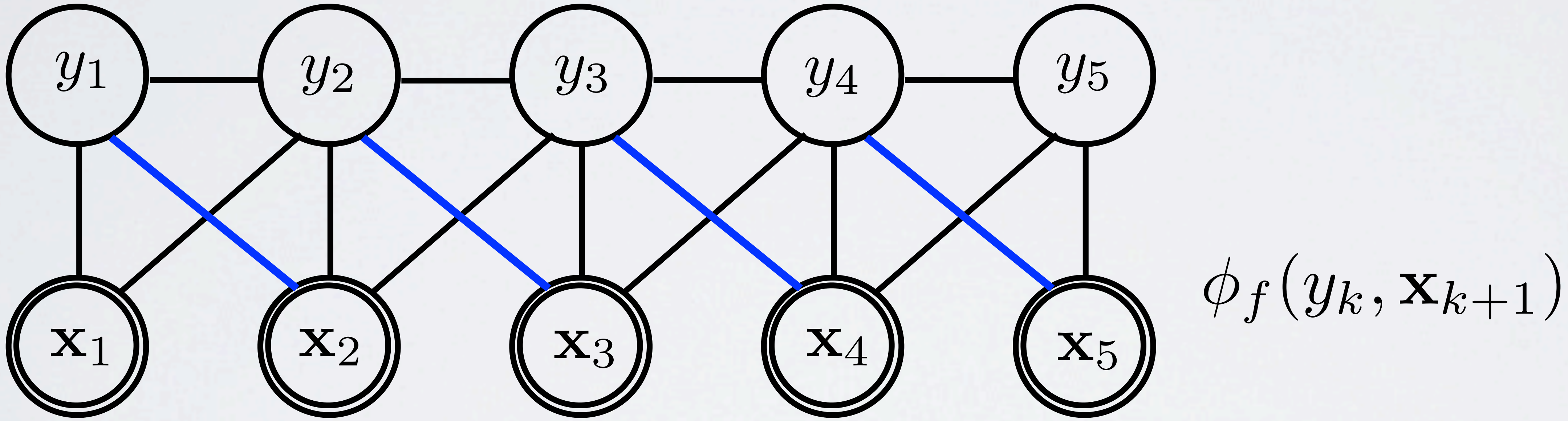

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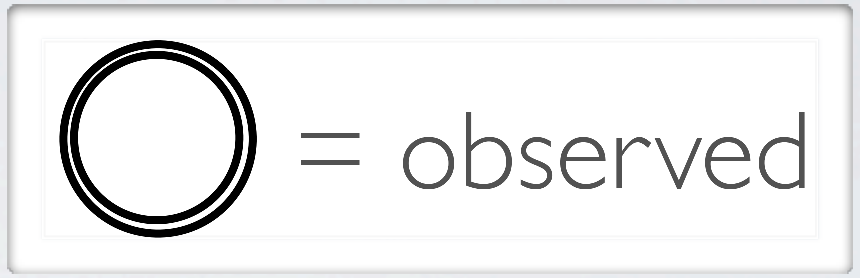

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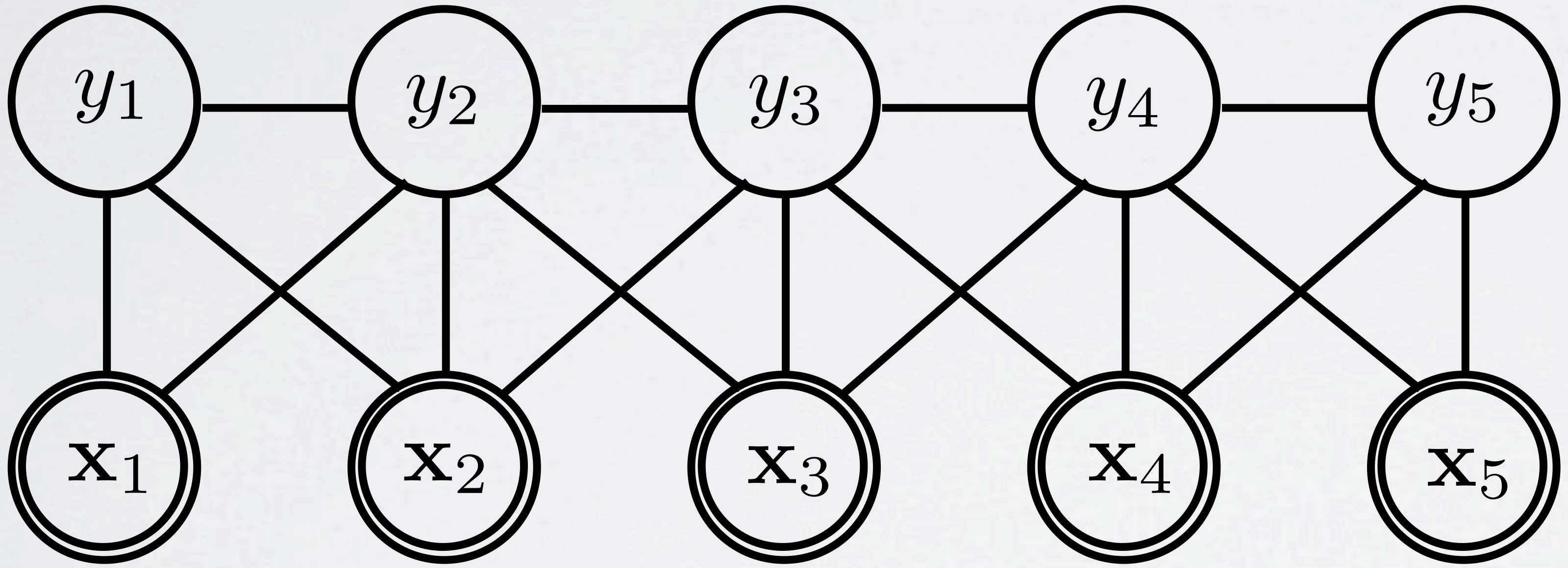

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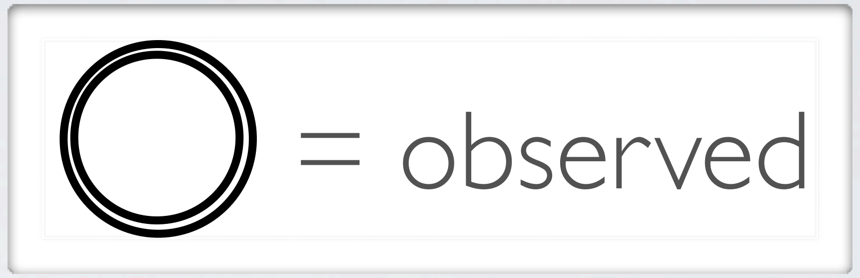

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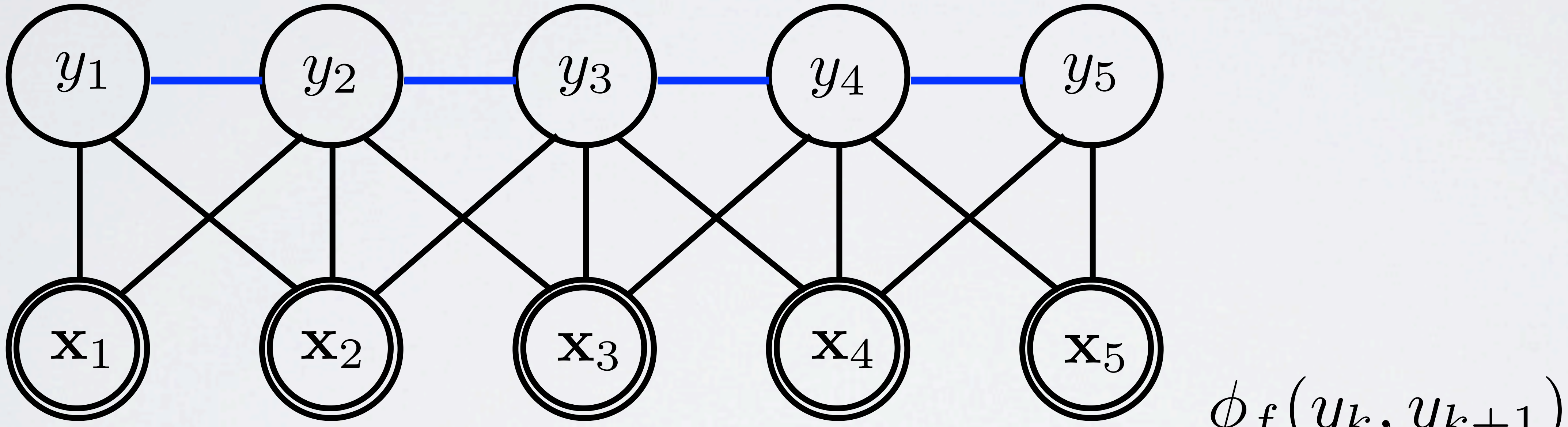

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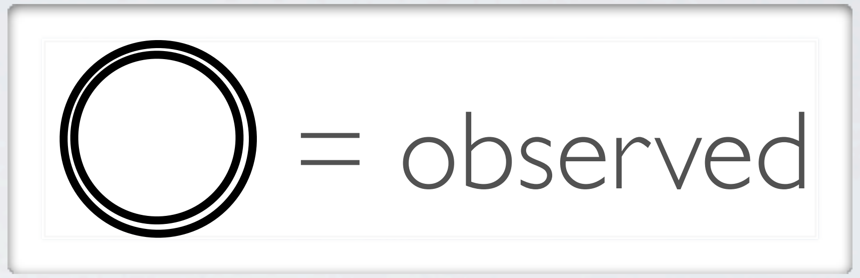

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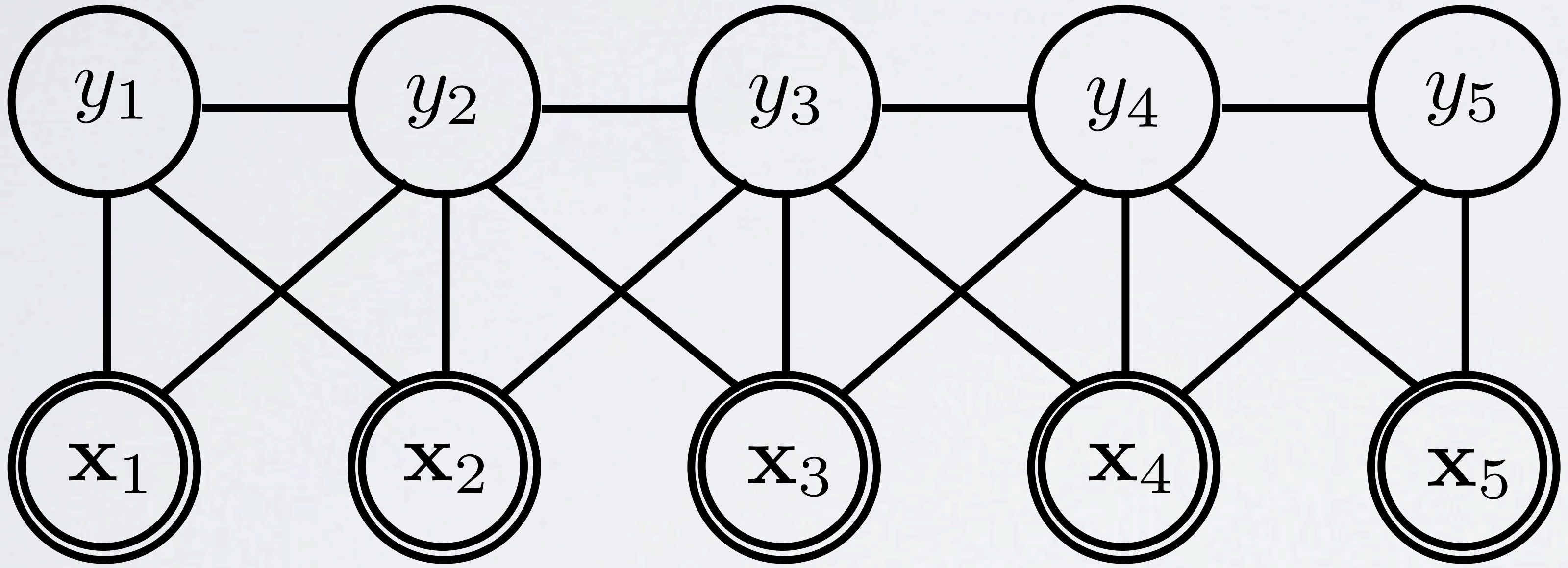

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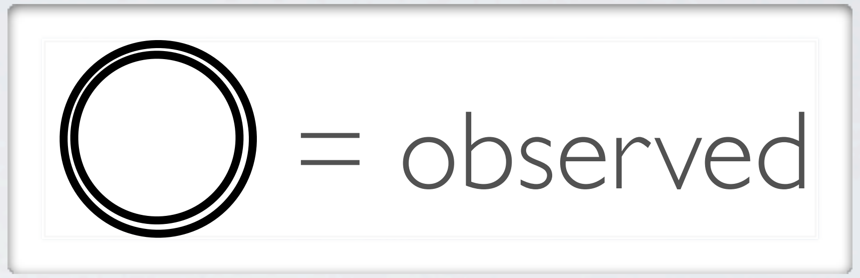

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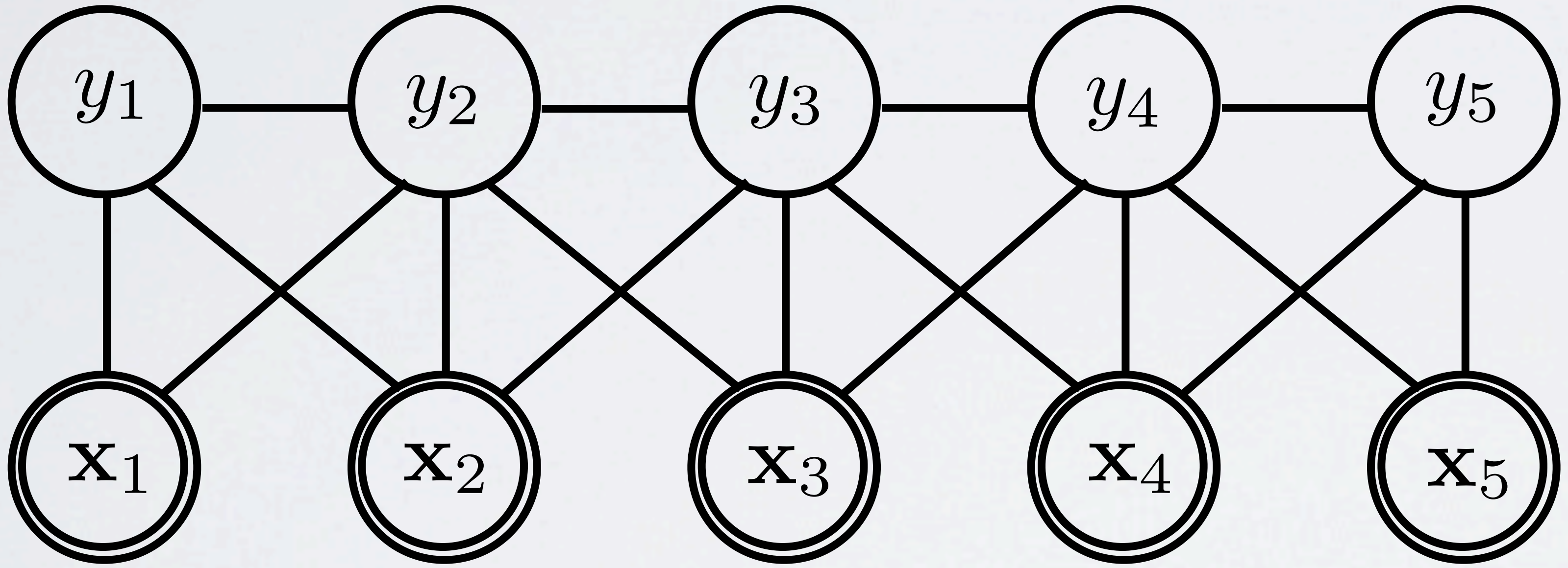

- Local Markov property
  - each node is independent of other nodes **given** its neighbors

MARKOV NETWORK VISUALIZATION

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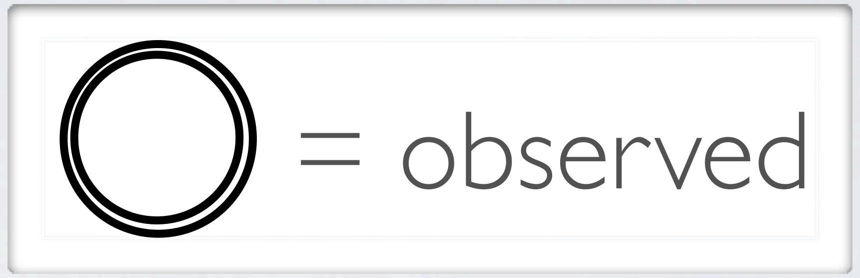

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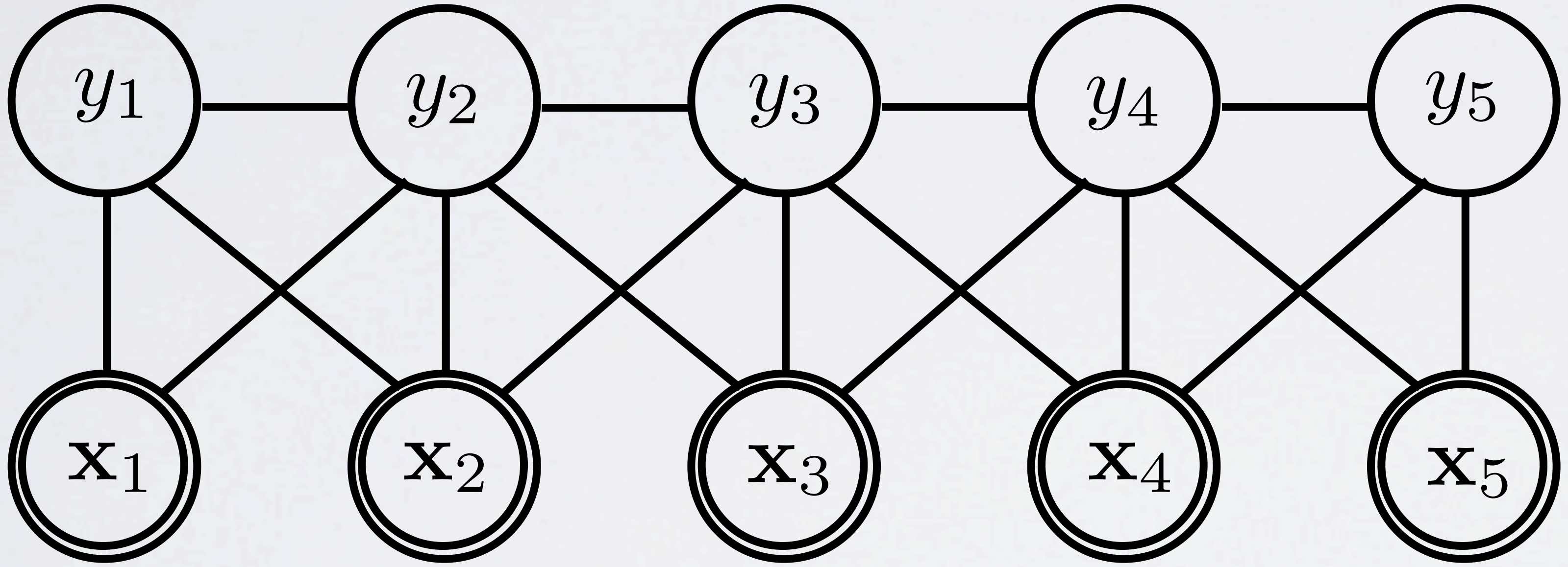
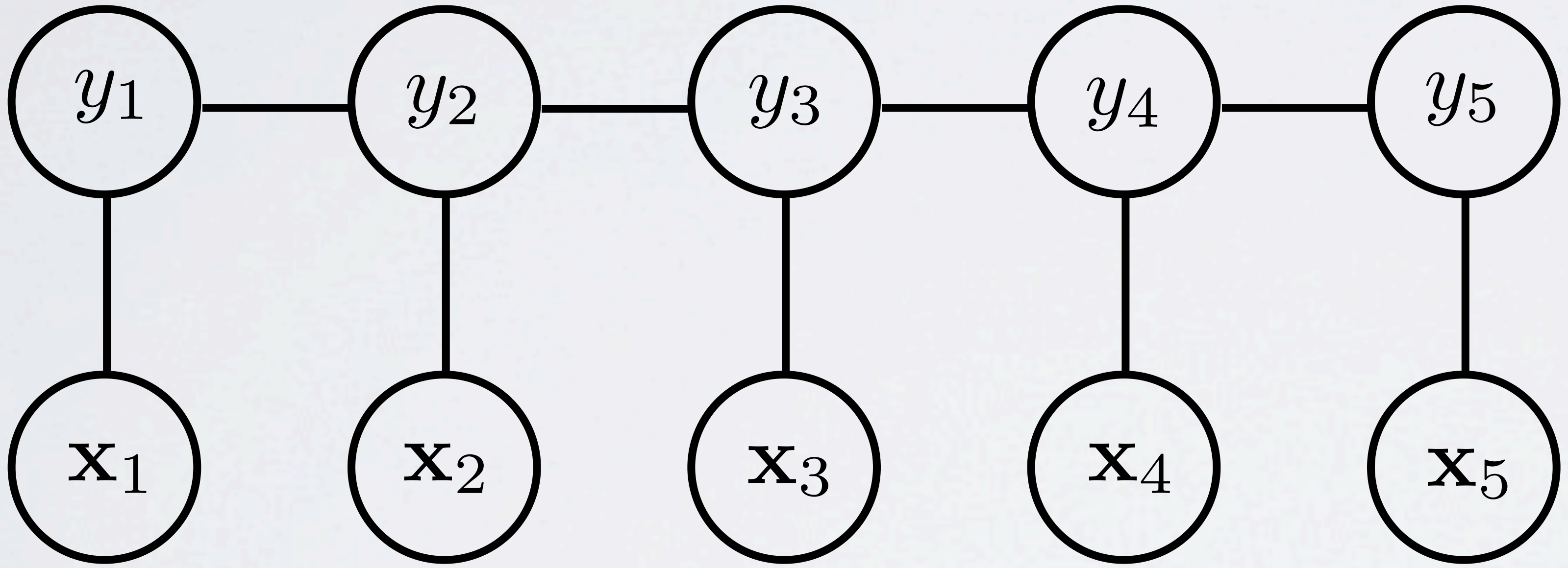


- General conditional independence
  - two nodes are conditionally independent if all paths between them contain at least one of the conditioning node

DIRECTED VS. UNDIRECTED

Topics: undirected graphical model, directed graphical model

- CRF is an undirected graphical model

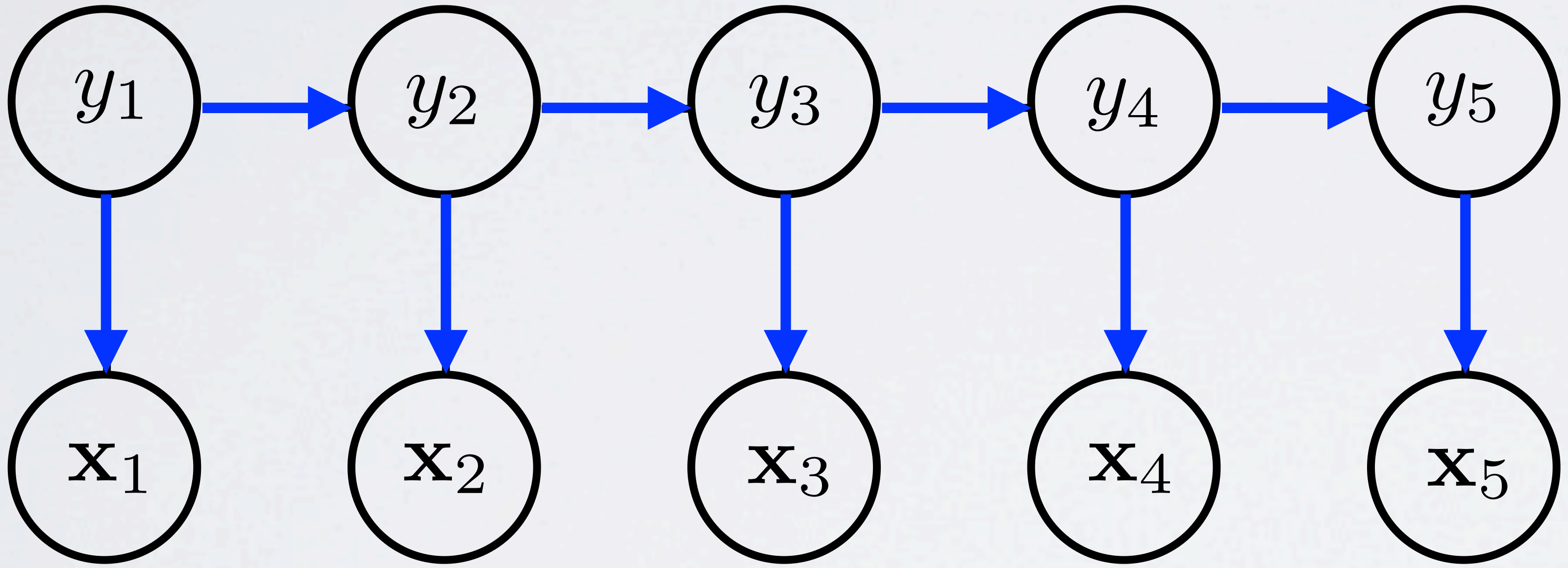


$$\left.\begin{array}{l} \phi_f(y_k, \mathbf{x}_k) \\ \\ \phi_f(y_k, y_{k+1}) \end{array}\right\} \text{only need to be non-negative}$$

DIRECTED VS. UNDIRECTED

Topics: undirected graphical model, directed graphical model

- HMM is a directed graphical model



$$\phi_f(y_k, \mathbf{x}_k)$$

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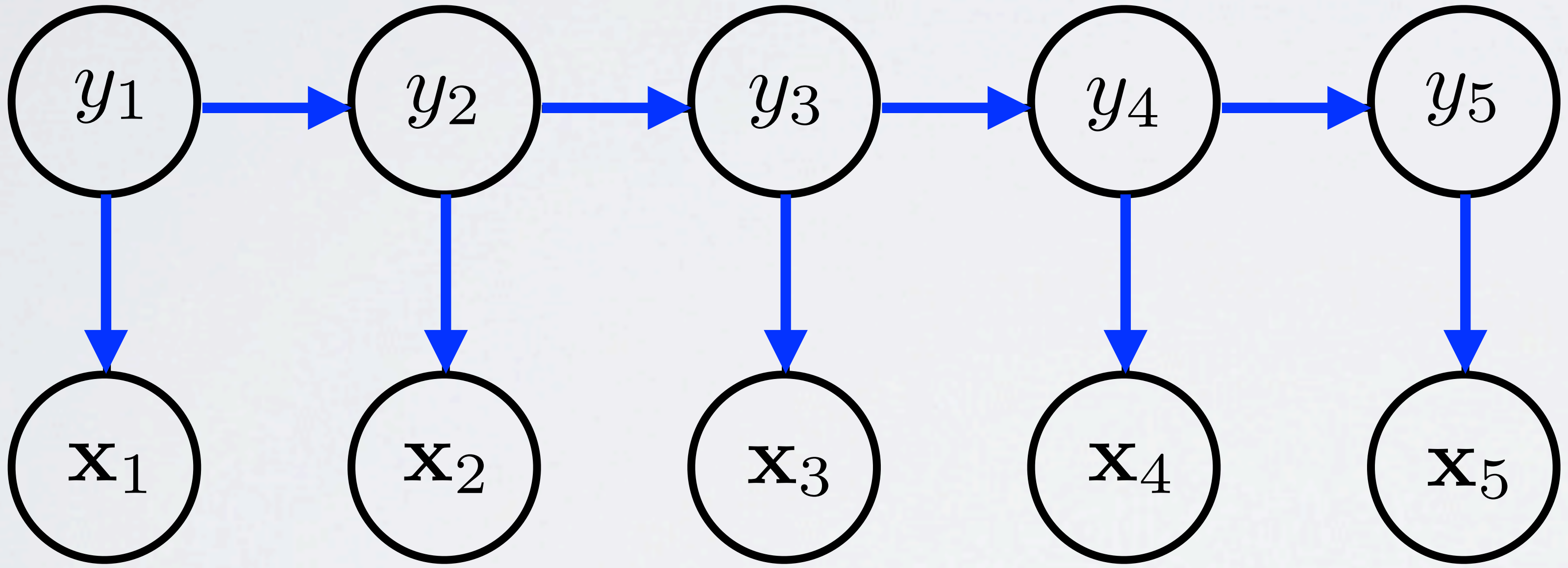

$$\phi_f(y_k, \mathbf{x}_k) = p(\mathbf{x}_k | y_k)$$

$$\phi_f(y_k, y_{k+1}) = p(y_{k+1} | y_k)$$