Neural networks

Sparse coding - definition

UNSUPERVISED LEARNING

Topics: unsupervised learning

- Unsupervised learning: only use the inputs $\mathbf{x}^{(t)}$ for learning
  - automatically extract meaningful features for your data
  - leverage the availability of unlabeled data
  - add a data-dependent regularizer to trainings

- We will see 3 neural networks for unsupervised learning
  - restricted Boltzmann machines
  - autoencoders
  - **sparse coding model**

SPARSE CODING

Topics: sparse coding

- For each $\mathbf{x}^{(t)}$ find a latent representation $\mathbf{h}^{(t)}$ such that:
  - it is sparse: the vector $\mathbf{h}^{(t)}$ has many zeros
  - we can reconstruct the original input $\mathbf{x}^{(t)}$ as well as possible
- More formally:

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\, \mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$$

  - we also constrain the columns of $\mathbf{D}$ to be of norm 1
    - otherwise, $\mathbf{D}$ could grow big while $\mathbf{h}^{(t)}$ becomes small to satisfy the prior
  - sometimes the columns are constrained to be no greater than 1

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sparsity penalty

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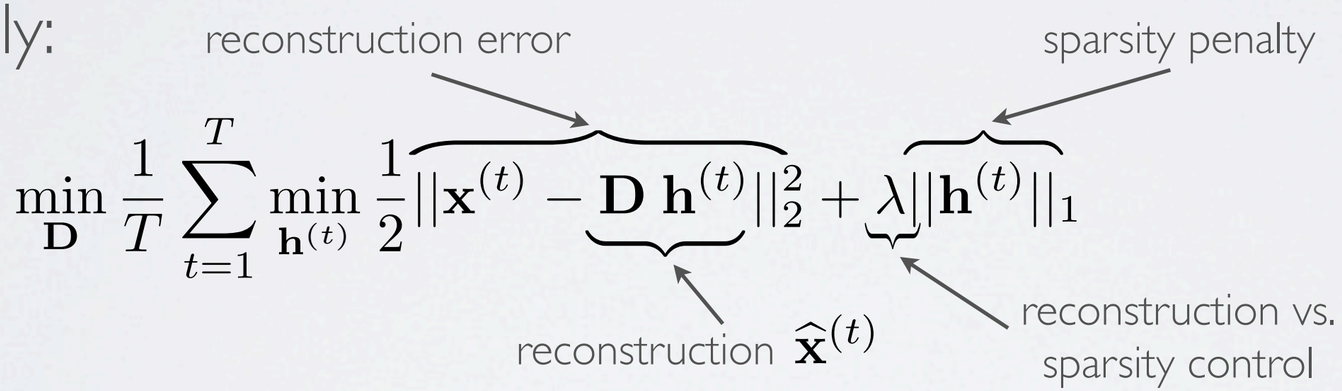

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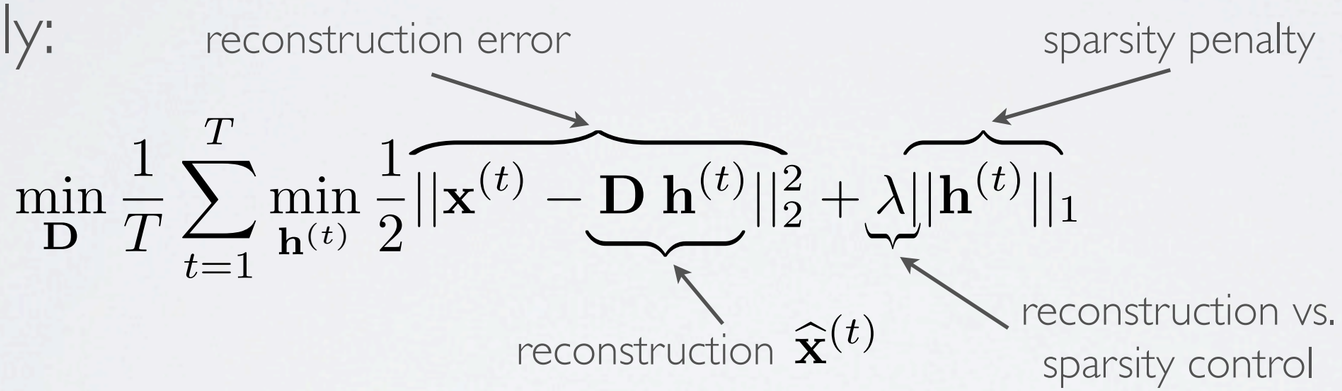
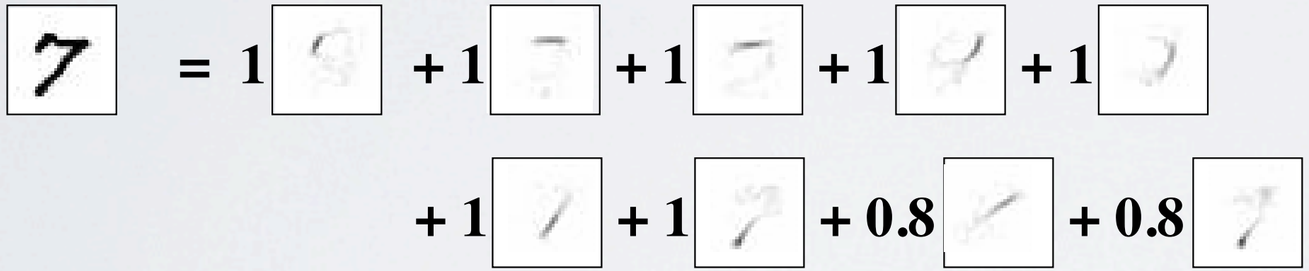


  - $\mathbf{D}$ is equivalent to the autoencoder output weight matrix
  - however, $\mathbf{h}(\mathbf{x}^{(t)})$ is now a complicated function of $\mathbf{x}^{(t)}$
    - encoder is the minimization $\mathbf{h}(\mathbf{x}^{(t)}) = \arg\min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$

SPARSE CODING

Topics: dictionary

- Can also write $\widehat{\mathbf{x}}^{(t)} = \mathbf{D}\,\mathbf{h}(\mathbf{x}^{(t)}) = \sum_{\substack{k \text{ s.t.} \\ h(\mathbf{x}^{(t)})_k \neq 0}} \mathbf{D}_{.,k}\, h(\mathbf{x}^{(t)})_k$



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  - in certain applications, we know what dictionary matrix to use
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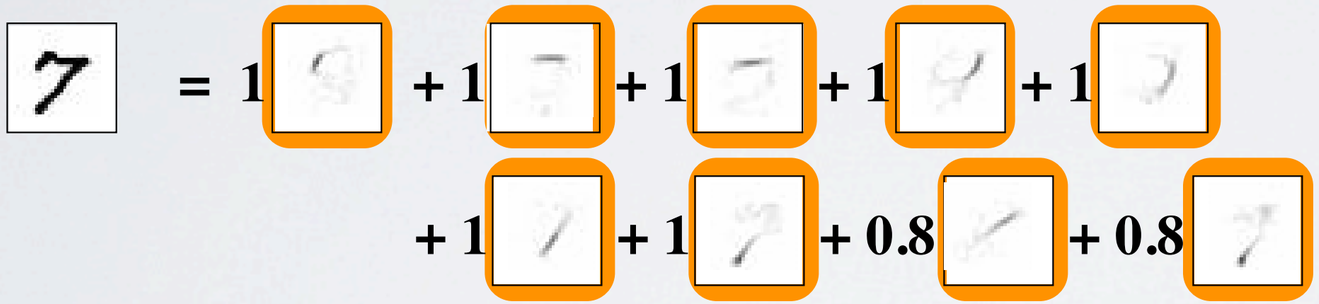

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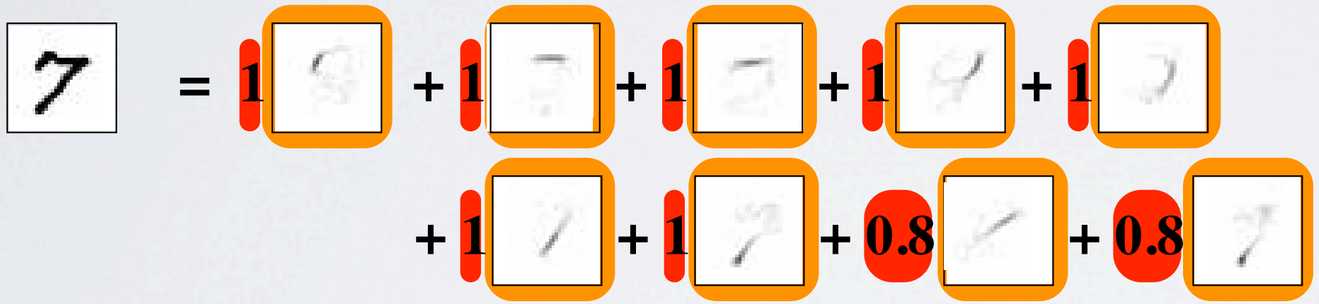

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